647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
Feb/Mar. -2021 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

1(A). Answer any two. **[10]**

(1) R is an equivalence relation on set X then prove that

(i) $\cup_{x \in X}[x]$ = X

(ii) xRy $\Leftrightarrow [x] = [y]$

(2) State and prove distributive inequalities.

(3) Let (L, $*$, $\oplus$) be a Lattice, a, b, c $\in$ L then prove that

a $*$ (b $\oplus$ c) = (a $*$ b) $\oplus$ (a $*$ c) $\Leftrightarrow$ a $\oplus$ (b $*$ c) = (a $\oplus$ b) $*$ (a $\oplus$ c)

1(B). Answer any one. **[4]**

(1) In usual notation show that (S_n, D) is poset.

(2) Define: Minimal member. Also find minimal member of (S, D) where S = {2, 3, 6, 12, 24, 36}.

2(A). Answer any two. **[10]**

(1) If (B, $*$, $\oplus$, ', 0, 1) is Boolean algebra and a, b $\in$ B then prove that

a $\leq$ b $\Leftrightarrow$ a $*$ b' = 0 $\Leftrightarrow$ b' $\leq$ a' $\Leftrightarrow$ a' $\oplus$ b = 1.

(2) If S = {a, b, c}, B = {0, 1} then show that f: P(S) $\rightarrow$ B is Boolean homomorphism for P(S), $\cap$, $\cup$, ^{c}, ϕ , S) and (B, $*$, $\oplus$, ', 0, 1) where

f(x) = 1, b $\in$ X

f(x) = 0, b $\notin$ X

(3) If (B, $*$, $\oplus$, ', 0, 1) is Boolean algebra and x_1, x_2 $\in$ B then prove that A($x_1 \oplus x_2$) = A(x_1) A(x_2).

2 (B). Answer any one. **[4]**

(1) In usual notation prove that (a $*$ b)' = a' $\oplus$ b'.

(2) Simplify following Boolean sentence.

(a'$*$ b' $*$ c) $\oplus$ (a $*$ b' $*$ c) $\oplus$ (a $*$ b' $*$ c')

3(A). Answer any two. **[10]**

(1) In usual notation obtain Cauchy – Riemann conditions in Cartesian form.

(2) Find an analytic function f (z) = u + i v such that u – v = x + y.

(3) Prove that f (z) = $\frac{\overline{(z)}^2}{z}$, z $\neq$ 0

And f (z) = 0 , z = 0 satisfied C – R conditions at origin however f(z) is not analytic at origin.

3(B). Answer any one. **[4]**

(1) Prove that u = $r^2\cos 2\theta$ is a harmonic function and find it's conjugate.

(2) Obtain Laplace equation in polar form.

4(A). Answer any two. **[10]**

(1) State and prove Cauchy's integral formula.

(2) Find $\int_c \frac{z}{(9-z^2)(z+i)}$ dz; where C: $|z|$ = 2.

(3) Show that $\left|\int_c \frac{z+4}{z^3-1}\, dz\right| \leq \frac{6\pi}{7}$ Where C be the arc of the circle $|z|$ = 2 from z = 2 to z = 2i.

4(B). Answer any one. **[4]**

(1) In usual notation prove that $\left|\int_c f(z)dz\right| \leq$ ML.

(2) Find: $\int_c \frac{dz}{(z-3)}$ where C: | z - 2| = 5.

5(A). Answer any two. **[10]**

(1) State and prove the fundamental theorem of algebra.

(2) State and prove Liouville's theorem.

(3) State and prove Morera's theorem.

5(B). Answer any one. **[4]**

(1) Find: $\int_c \frac{5z^2-7z+3}{(z-1)^2}$ dz where C: |z| = 2

(2) If C: |z| = 3 is the closed contour and the integral is fallen in positive sense and g(z) = $\int_c \frac{2s^2-s-2}{s-z}$ ds then prove that g(2) = 8πi and find g(z) when |z| $>$ 3.

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